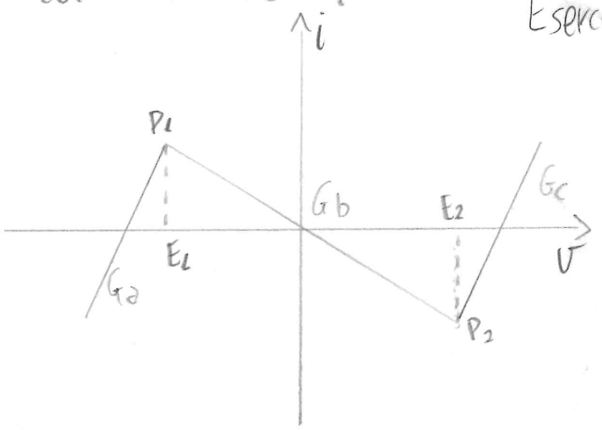
Reti non lineari



Esercizio 1

La corrente "i" é costituita da 3 contributi:

$$i = i_0 + i_1 + i_2$$

$$i_0 = G_a(v - E_1) + \underbrace{G_b E_1}_{\to \text{Traslazione!}}$$

$$i_1 = \frac{1}{2} G_1 \left[|v - E_1| + (v - E_1) \right] \to \text{Concave Resistor}$$

$$i_2 = \frac{1}{2} G_2 \left[|v - E_2| + (v - E_2) \right]$$

Dobbiam avere $\begin{cases} G_a + G_1 = G_b \\ G_a + G_1 + G_2 = G_c \end{cases}$ $\begin{Bmatrix} G_1 \\ e \\ G_2 \end{Bmatrix} \to$ "pendenze" dei concave resistors!

Si scrive dunque la caratteristica in termini di conduttanze, usando sostanzialmente concave resistors e resistori "normali"!

Calcoliamo a questo punto G_1 e G_2, ossia le "pendenze" dei concave!

$$\to G_1 = G_b - G_a$$
$$G_2 = G_c - G_a - G_1 = G_c - G_a - G_B + G_a = G_c - G_B$$

Possiamo a questo punto calcolare "i", sostituendo!

$$i = i_0 + i_1 + i_2 = G_a(v - E_1) + G_b E_1 + \frac{1}{2}(G_b - G_a)\left[|v - E_1| + (v - E_1)\right] + \frac{1}{2}(G_c - G_B)\left[|v - E_2| + (v - E_2)\right] =$$

$$= E_1(G_b - G_a) + G_a v + \frac{1}{2}(G_B - G_a)|v - E_1| + \frac{1}{2}(G_B - G_a)v - \frac{1}{2}(G_B - G_a)E_1 + \frac{1}{2}(G_c - G_b)|v - E_2| + \frac{1}{2}(G_c - G_B)v - \frac{1}{2}E_2(G_c - G_B)$$

$$= v\left[G_a + \frac{1}{2}(\cancel{G_B} - G_a) + \frac{1}{2}(G_c - \cancel{G_B})\right] + \frac{1}{2}(G_B - G_A)E_1 - \frac{1}{2}(G_c - G_B)E_2 + \frac{1}{2}(G_B - G_A)|v - E_1| + \frac{1}{2}(G_c - G_B)|v - E_2|$$

$$= \frac{1}{2}\left[(G_B - G_A)|v - E_1| + (G_c - G_B)|v - E_2|\right] + \frac{1}{2}v(G_c + G_A) + \frac{1}{2}E_1(G_B - G_A) - \frac{1}{2}E_2(G_c - G_B)$$

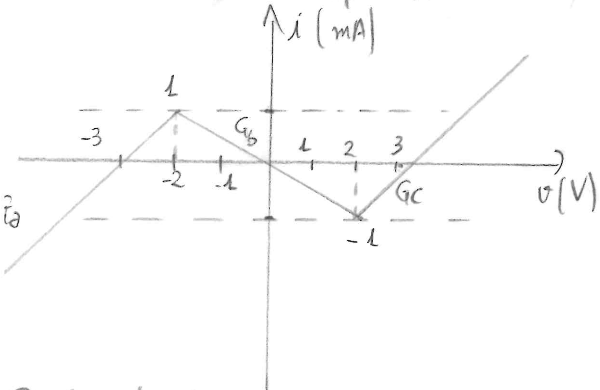
Imponendo la simmetria rispetto all'origine, $\begin{cases} E_1 = -E \\ E_2 = E \\ G_a = G_c \end{cases}$

$$\to G_a v + \frac{1}{2}(G_b - G_a)\left[|v + E| - |v - E|\right]$$

Reti non lineari

Es. 2: uguale all' "1", "simmetrico! 1 1"

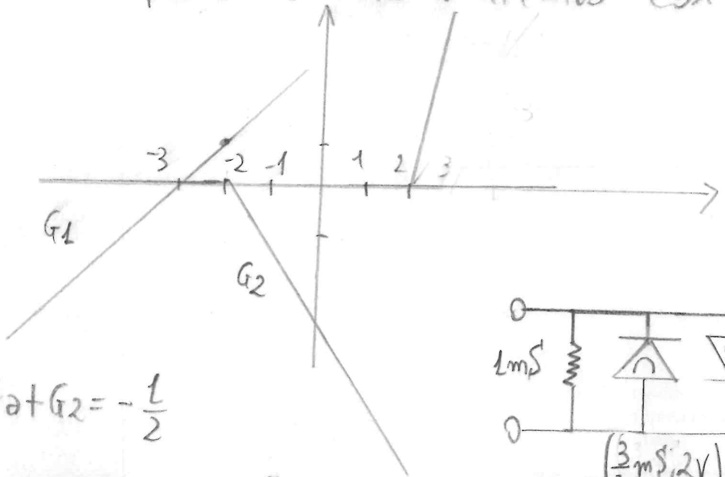
Es. 3: sintetizzare, con resistori e concave resistors, la seguente caratteristica:



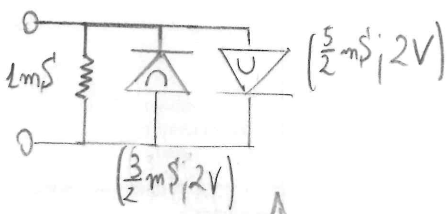
Come si fa?

Beh, vediamo che: $\begin{cases} G_a = 1 \\ G_b = -\frac{1}{2} \\ G_c = 1 \end{cases}$

Potremmo fare il circuito così:



Partendo da G_a, si fa in modo, coi concave, di ottenere questa cosa!



È ok? :-)

Quindi, $G_a = 1$; $G_b = -\frac{1}{2} \to G_a + G_2 = -\frac{1}{2}$

$\to G_2 = -\frac{3}{2}$; $G_3 + G_2 = 1 \to G_3 = 1 - G_2 = \frac{5}{2}$

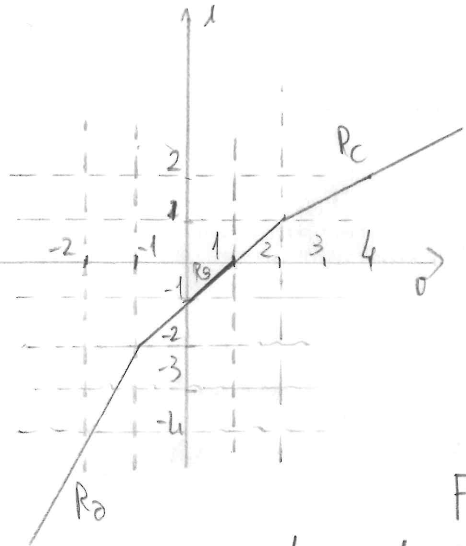
$$i = \underbrace{G_a\left(v-(-2)\right) + G_b\cdot(-2)}_{1} + \underbrace{\frac{1}{2} G_2 \left[\left|v-(-2)\right| + \left(v-(-2)\right)\right]}_{2} + \underbrace{\frac{1}{2} G_3 \left[\left|v-2\right| + (v-2)\right]}_{3}$$

$$= (v+2) + \left(-\frac{3}{2}\right)\frac{1}{2}\left[\left|v+2\right| + (v+2)\right] + \frac{1}{2}\left(\frac{5}{2}\right)\left[\left|v-2\right| + (v-2)\right]$$

...

Non sarebbe richiesta!

4a)



$R_a = \frac{1}{2}$; $R_a + R_b = 1$; $R_a + R_b + R_c = 2$

[Nota: il reciproco di prima. Se, ad es. nel primo tratto v aumenta di 1, i aumenta di 2, ma $R = \frac{v}{i} = \frac{1}{2}$! :-)]

Risolviamo il sistema e determiniamo le 3 variabili:

$R_a = 1$; $R_a + R_b = 1 \rightarrow R_b = \frac{1}{2}$; $R_c = 1$

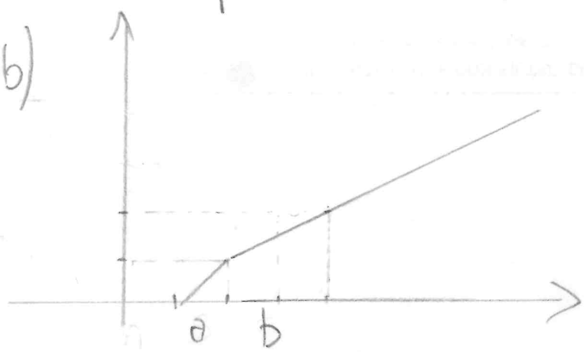
Facendo dunque $v = f(i)$:

$$v = \frac{1}{2} i + \frac{1}{2} \cdot \frac{1}{2} \cdot \left[|i+2| + (i+2) \right] + \frac{1}{2} \cdot 1 \cdot \left[|i-1| + (i-1) \right] =$$

$$= i \left(\frac{1}{2} + \frac{1}{4} + \frac{1}{2} \right) + \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}} + \frac{1}{4} |i+2| + \frac{1}{2} |i-1| = \frac{5}{4} i + \frac{1}{4} |i+2| + \frac{1}{2} |i-1|$$

Domanda: perché con le "i" stavolta? :-) $\rightarrow$ In questo caso, i o v è indifferente! Funzione biiettiva!

4b)



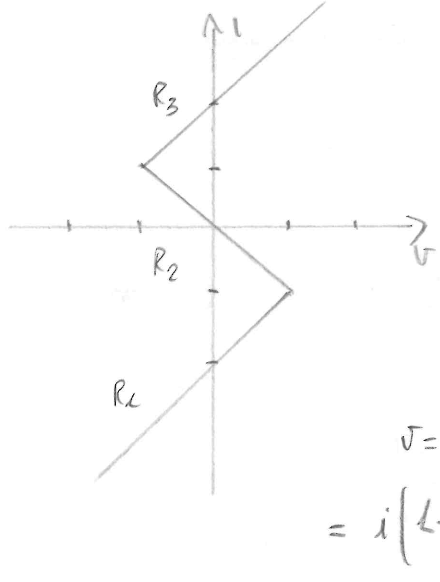
$G_a = 1$; $G_a + G_b = \frac{1}{2}$; $\rightarrow G_b = -\frac{1}{2}$.

$$i = i_a + i_b = \frac{1}{2} \cdot 1 \left[|v-1| + (v-1) \right] + \frac{1}{2} \cdot \left(-\frac{1}{2} \right) \cdot \left[|v-2| + (v-2) \right]$$

$$i = v \left(\frac{1}{2} - \frac{1}{4} \right) - \frac{1}{2} + \frac{1}{2} + \frac{1}{2} |v-1| - \frac{1}{4} |v-2| =$$

$$= \frac{1}{4} v + \frac{1}{2} |v-1| - \frac{1}{4} |v-2|$$

4c)



Chieder conferma, ma credo che qui lavorar con le "i" sia di obbligo! :-)

Vediamo dunque che:

$R_1 = 1$; $R_2 = -1$; $R_3 = 1$.

$$\begin{cases} R_A = 1 \\ R_B + R_A = -1 \\ R_B + R_A + R_C = 1 \end{cases} \Rightarrow \begin{cases} R_A = 1 \\ R_B = -2 \\ R_C = 2 \end{cases}$$

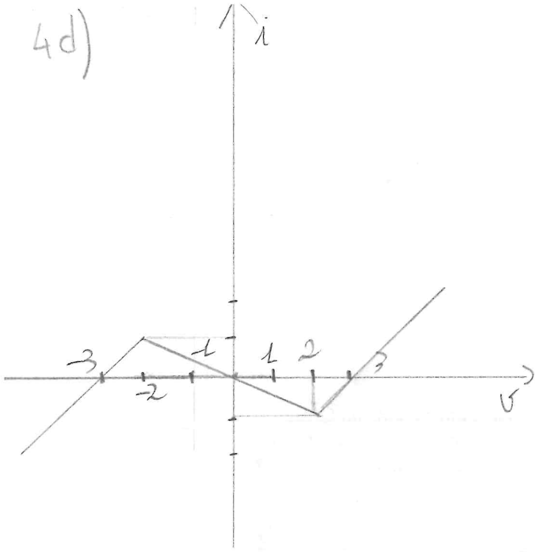
$\Downarrow$

$$v = i + \frac{1}{2} \cdot (-2) \cdot \left[|i+1| + (i+1) \right] + \frac{1}{2} \cdot 2 \cdot \left[|i-1| + (i+1) \right]$$

$$= i(1 - 1 + 1) \cancel{-1} \cancel{+1} + \ldots = i - |i+1| + |i-1|$$

7/1/08: non ho fatto ancora il 4d, 4e, 5

4d)



$G_A = 1$; $G_B = -\frac{1}{2}$; $G_C = 1$

$$i = G_A v + \frac{1}{2}\left(|v+2| + (v+2)\right)G_2 + \frac{1}{2}\left(|v-2| + (v-2)\right)G_3$$

Se $G_A = 1$, $G_B = G_A - G_2 = -\frac{3}{2}$;

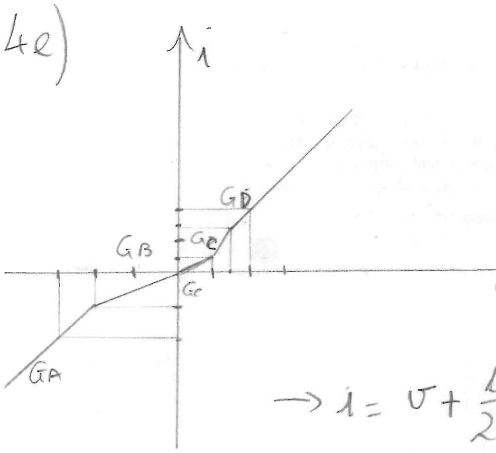
$G_C = 1 \rightarrow G_3 = +\frac{3}{2} + 1 = \frac{5}{2}$

$$i = G_A(v - E_1) + G_B E_1 + \frac{1}{2}\left[|v-E_1| + (v-E_1)\right](G_b - G_a) + \frac{1}{2}(G_c - G_b)\left[|v-E_2| + (v-E_2)\right]$$

Si ricordi che, in caso di transcaratteristiche simmetriche rispetto origine,

$$i = G_a \cdot v + \frac{1}{2}(G_b - G_a)\left(|v+E| - |v-E|\right) = v + \frac{1}{2}\left(-\frac{1}{2} - 1\right)\left[|v+2| - |v-2|\right]$$

4e)



$$i = vG_A + G_B E_1 + \frac{1}{2}(G_B - G_A)\left[|v-E_1| + (v-E_1)\right] + \frac{1}{2}(G_C - G_B)\left[|v-E_2| + (v-E_2)\right] + \frac{1}{2}(G_D - G_C)\left[|v-E_3| + (v-E_3)\right]$$

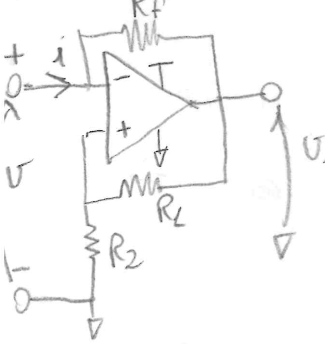
$G_A = 1$; $G_B = \frac{1}{2}$; $G_C = 2$; $G_D = 1$

$$\rightarrow i = v + \frac{1}{2}\cdot(-2) + \frac{1}{2}\left(-\frac{1}{2}\right)[\ldots] + \frac{1}{2}\left(\frac{3}{2}\right)[\ldots] + \frac{1}{2}(-1)[\ldots]$$

$$= v\left(1 - \frac{1}{4} + \frac{3}{4} - \frac{1}{2}\right) + 1 - \frac{1}{2} + \cancel{\frac{3}{4}} - \cancel{\frac{1}{2}\cdot\frac{3}{2}} + \ldots$$

$$= v + \frac{1}{2} - \frac{1}{4}|v+2| + \frac{3}{4}|v-1| - \frac{1}{2}\left|v - \frac{3}{2}\right|$$

5) Studiare la caratteristica (transresistenza $\frac{v}{i}$) del seguente bipolo

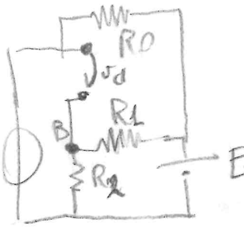


$$i = \frac{v - v_u}{R_f};$$

$$v_u \frac{R_2}{R_1 + R_2} = v \rightarrow v_u = v\left(1 + \frac{R_1}{R_2}\right)$$

$$\rightarrow i = \frac{\cancel{v} - \left(\cancel{1} + \frac{R_1}{R_2}\right)v}{R_f} = -\frac{R_1}{R_f R_2} v \rightarrow \frac{v}{i} = R = -\frac{R_f R_2}{R_1}$$

Studio saturazione

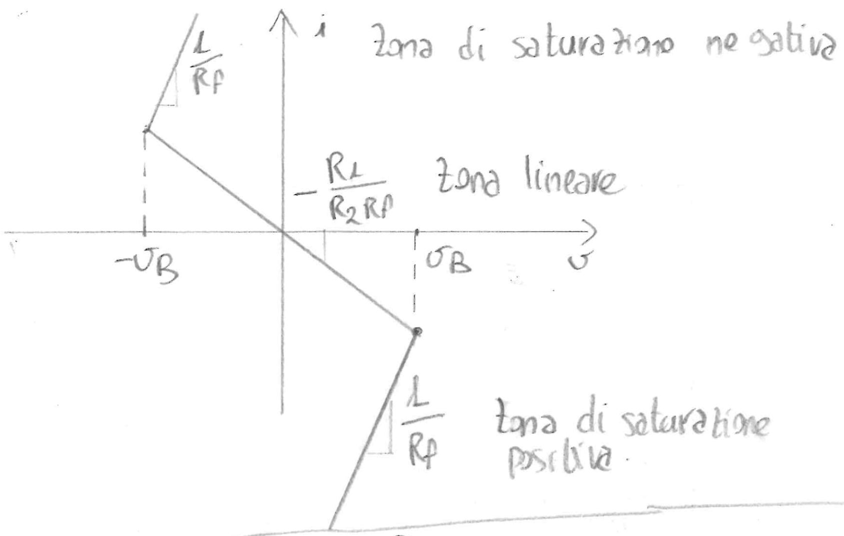


$\leftarrow$

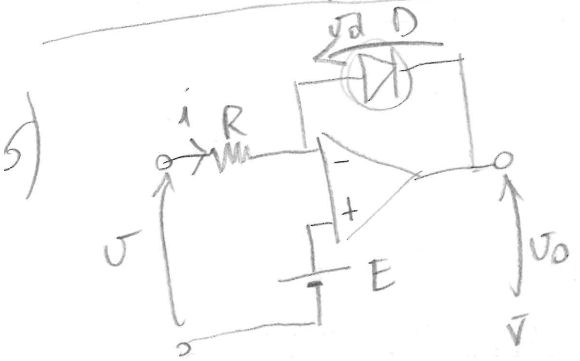
Si resta in linearità fino a quando:

$$v = \frac{R_2}{R_1 + R_2} v_u < \underbrace{\frac{R_2}{R_1 + R_2} E_S}_{\text{saturazione}} \rightarrow |v| < \underbrace{\frac{R_2}{R_1 + R_2} E_S}_{V_B}$$

$$i = \frac{v - E_S}{R_f}, \; v_d > 0; \quad v_d > 0 = (v_+ - v_-) > 0$$
$$= v < v_B$$



5)



Data $E < E_s$, si ha che:

$v = Ri + E$

Come si sa, l'operazionale é in regione lineare se:

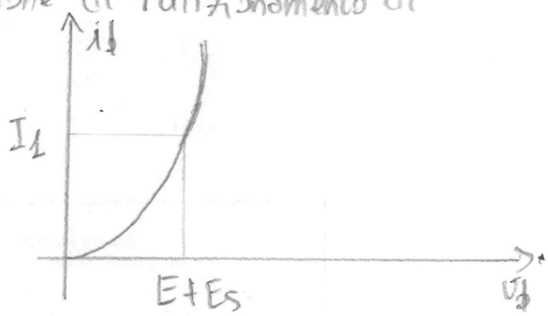
$|v_d| < E_s \quad [-E_s < v_0 < E_s]$

Ma $v_0 = E - \underbrace{v_D}_{\text{Tensione diodo!}}$

$\rightarrow E - E_s < v_d < E + E_s \rightarrow$

Si sa che l'equazione di funzionamento di un diodo é:

$I_D = I_S \left(e^{\frac{v_d}{V_T}} - 1\right)$

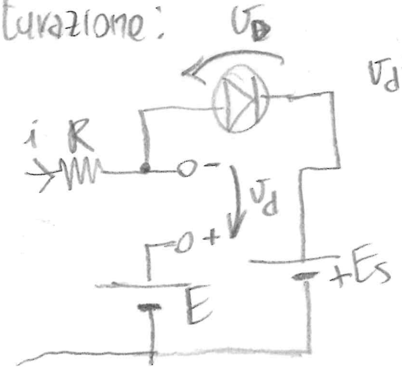


Dal momento che $i = I_D$, il circuito funzionerá in regione lineare se:

$0 \leq i < I_1$; poiché $v = Ri + E$,

$\rightarrow E \leq v < RI_1 + E$

Saturazione:



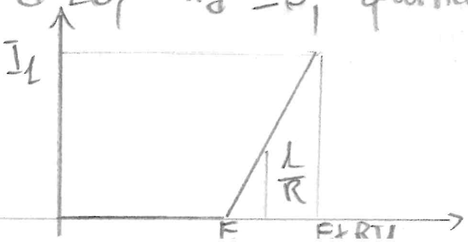
$v_d > 0 \Rightarrow E - (v - Ri) = E - v + Ri > 0$

$\rightarrow v < E + Ri$

$v_D = v - Ri - E_s < (E + Ri) - Ri - E_s$

$\rightarrow E - E_s < 0$

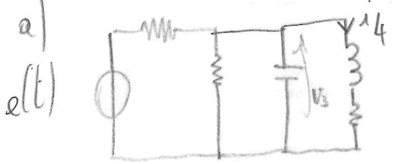
Essendo $v_d < 0$, $i_d \simeq 0$, quindi la resistenza é infinita, la conduttanza nulla.



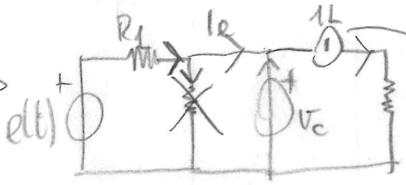
Reti non lineari - Esercitazione 2

1) Scriviamo le eq. di stato dei seguenti circuiti:

a)



$$v_L = L\frac{d i_L(t)}{dt} \quad ; \quad i_C = C\frac{d v_C(t)}{dt}$$

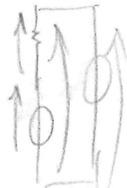


$\boxed{v_L = v_C - i_L R_3}$ ok

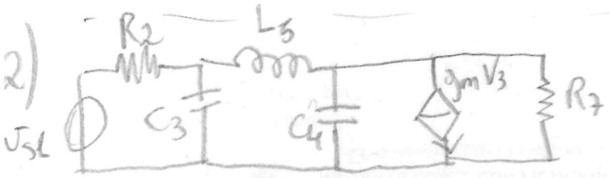
$$i_e = \frac{v_C - e(t)}{R_1} - \frac{v_C}{R_2}$$

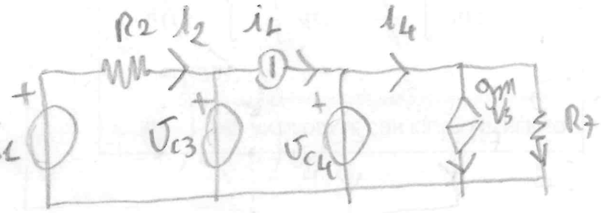
$\rightarrow \frac{1}{2} v_C - 5 + i_L$

?



2)





$\rightarrow \quad i_4 = \frac{v_{C4}}{R_7} + g_m v_{C3} \qquad i_{C4} = i_L - i_4 \; ;$

$i_2 = \frac{v_{S1} - v_{C3}}{R_2} \; ; \; i_{C3} = i_2 - i_L$

$v_L = v_{C3} - v_{C4}$

Ricorda:

$L \rightarrow v = L\frac{di_L}{dt}$

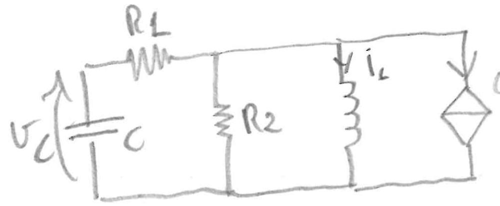
$C \rightarrow i = L\frac{dv_C}{dt}$

$$\frac{di_L}{dt} = \frac{v_{C3}}{L_5} - \frac{v_{C4}}{L_5} \qquad \frac{dv_3}{dt} = \frac{v_{S1}}{C_3R_2} - \frac{v_{C3}}{C_3R_2} - \frac{i_L}{C_3}$$

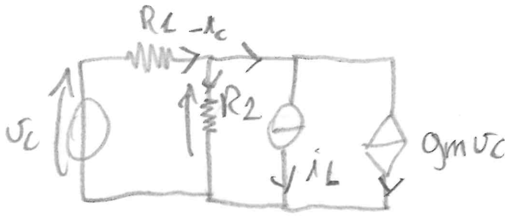
$$\frac{dv_4}{dt} = \frac{i_L}{C_4} - \frac{v_{C4}}{C_4R_7} - \frac{g_m v_{C3}}{C_4}$$

$$\rightarrow \begin{pmatrix} \frac{dv_3}{dt} \\ \frac{dv_4}{dt} \\ \frac{di_L}{dt} \end{pmatrix} = \begin{pmatrix} \frac{1}{L_5} & -\frac{1}{L_5} & 0 \\ -\frac{g_m}{C_4} & -\frac{1}{C_4R_7} & +\frac{1}{C_4} \\ \frac{1}{L_5} & 0 & -\frac{1}{L_5} \end{pmatrix}$$

c)



$\longrightarrow$

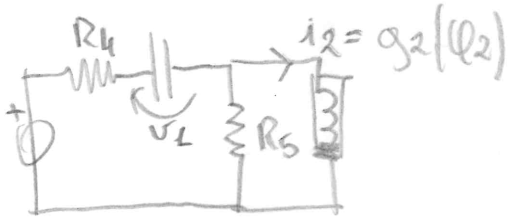


$$\longrightarrow \frac{\frac{v_C}{R_1}+0-i_L-g_m v_C}{\frac{1}{R_1}+\frac{1}{R_2}} = v_{R2} \longrightarrow v_{R2}\left(\frac{R_1+R_2}{\cancel{R_1}R_2}\right) = \frac{v_C}{\cancel{R_1}} - \frac{i_L R_1 - g_m v_C R_1}{\cancel{R_1}}$$

$$\longrightarrow v_{R2}\left(1+\frac{R_1}{R_2}\right) = v_C - i_L R_1 - g_m v_C R_1$$

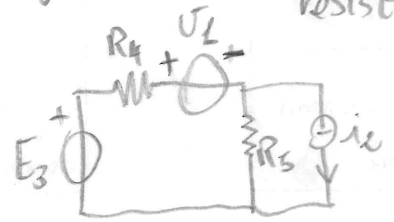
$$-i_C = \frac{v_{R2}}{R_2} + i_L + g_m v_C = \frac{1}{\cancel{R_2}}\left(v_C - i_L R_1 - g_m v_C R_1\right)\frac{\cancel{R_2}}{R_1+R_2} + i_L + g_m v_C$$

Sistemi non lineari



Variabili di stato: v_1 e φ_2

Passo al "circuito resistivo":



$$i_1 = C_1\frac{dv_1}{dt}; \quad v_2 = \frac{d\varphi_2}{dt}$$

$$\longrightarrow \frac{\frac{E_3-v_1}{R_4}+0-i_2}{\frac{1}{R_4}+\frac{1}{R_5}} = V_5;$$

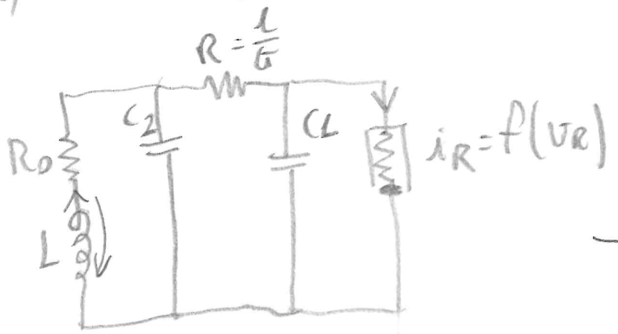
$$V_5\cdot\left(\frac{R_4+R_5}{\cancel{R_4}R_5}\right) = \frac{E_3-v_1-i_2R_4}{\cancel{R_4}}$$

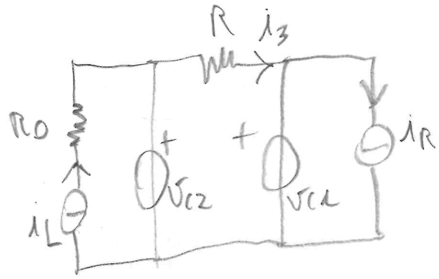
$$v_5 = v_2 = \frac{R_5}{R_4+R_5}\left[E_3-v_1-i_2R_4\right]$$

$$+i_1 = I_2 + \frac{v_5}{R_5}$$

$$\frac{dv_1}{dt} = -\frac{1}{C_1(R_4+R_5)}v_1 + \frac{R_5}{C_1(R_4+R_5)}g_2(\varphi_2) + \frac{1}{C_1(R_4+R_5)}e_3(t)$$

2) Oscillatore di Chua





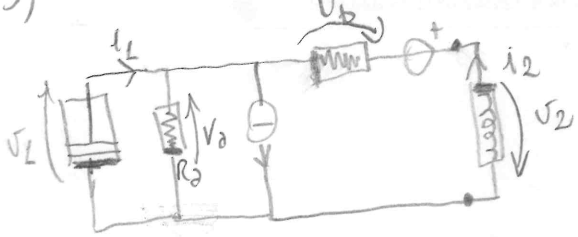
$i_3 = \dfrac{v_{C2} - v_{C1}}{R}$; $i_{C1} = i_3 - i_R$

$-v_L = R_0 i_L + v_{C2}$; $i_{C2} = i_L - i_3$

$\dfrac{dv_1}{dt} = \dfrac{1}{C_1}\left[(v_2 - v_1)G - f(v_1)\right]$; $\dfrac{dv_2}{dt} = \dfrac{1}{C_2}\left[i_L - \dfrac{v_{C2} - v_{C1}}{R}\right] = (-)\dfrac{1}{C_2}\left[(v_1 - v_2)G + i_L\right]$ e lui?! ...

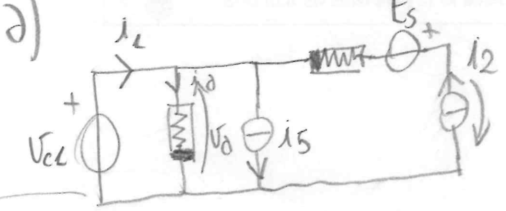
$\dfrac{di_L}{dt} = -\dfrac{1}{L}\left[v_2 + R_0 i_L\right]$

3) Circuito generale



$i_{C1} = -i_1$

a)



R_a: $i_a = v_a^5$; R_b: $v_b = e^{i_b}$ $\quad C_1$: $q_{C1} = v_{C1}^3$; L_2: $\varphi_{L2} = i_{L2}^{\frac{1}{3}}$

Scriver le eq. di sta to usando come var. v_{C1} e i_{L2}

$i_a = i_1 + (i_2 - i_3)$; $v_{L2} = E_S + v_b + v_{C1}$ $\qquad [v_a = v_{C1}] \to i_a = v_{C1}^5$

$i_1 = i_a + i_3 - i_b = v_1^5 - i_2 + I_S \to$ $\qquad v_{L2} = |E_S + e^{i_2} + v_1|$

$\to \begin{cases} i_{C1} = -v_{C1}^5 + i_{L2} - I_S \\ v_{L2} = -v_{C1} - e^{i_{L2}} - E_S \end{cases}$

Ora come var. di stato usiamo q_{C1} e φ_{L2}.
Si parte dalle seguenti equazioni:

$\begin{cases} i_{C1} = -v_{C1}^5 + i_{L2} - I_S \\ v_{L2} = -v_{C1} - e^{i_{L2}} - E_S \end{cases}$ $\qquad q_{C1} = v_{C1}^3 \to v_{C1} = \sqrt[3]{q_{C1}}$

$\qquad \varphi_{L2} = i_{L2}^{\frac{1}{3}} \to i_{L2} = \varphi_{L2}^3$

$i_{C1} = \dfrac{dq_1}{dt}$; $v_{L2} = \dfrac{d\varphi_2}{dt}$

$\to \begin{cases} \dfrac{dq_1}{dt} = -q_1^{\frac{5}{3}} + \varphi_2^3 - I_S \\ \dfrac{d\varphi_2}{dt} = -\sqrt[3]{q_1} - e^{\varphi_2^3} - E_S \end{cases}$

c) Partendo da: $\begin{cases} i_{C1} = -v_{C1}^3 + i_{L2} - I_S \\ v_{L2} = -v_{C1} - e^{i_{L2}} - E_S \end{cases}$ Calcolare le eq. di stato con v_{C1} e φ_{L2}

$\varphi_{L2} = \frac{dv_{L2}}{dt}$; $i_{C1} = C\frac{dv_{C1}}{dt}$

$\varphi_{L2} = i_{L2}^{\frac{1}{3}} \rightarrow i_{L2} = \varphi_{L2}^3$

$$\rightarrow \begin{cases} \frac{dv_{C1}}{dt} = \frac{1}{C(v)}\left[-v_{C1}^3 + \varphi_{L2}^3 - I_S\right] \\ \frac{d\varphi_{L2}}{dt} = -v_{C1} - e^{\varphi_{L2}^3} - E_S \end{cases}$$

Ma $C(v) = \frac{dq_{C1}}{dv} = 3v_{C1}^2$ [Condensatore non lineare!]

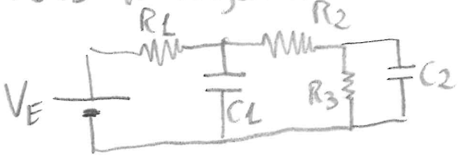
d) q_{C1} e i_{L2};

$q_{C1} = v_{C1}^3 \rightarrow v_{C1} = \sqrt[3]{q_{C1}}$; $i_{C1} = \frac{dq_1}{dt}$

$v_{L2} = L(i)\frac{di_{L2}}{dt}$ $\quad L(i) = \frac{d\varphi_{L2}}{di} = \frac{1}{3}\left(\frac{1}{\sqrt[3]{i_{L2}^2}}\right)$

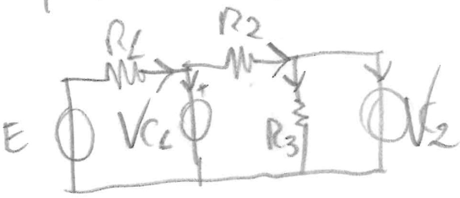
$$\rightarrow \begin{cases} \frac{dq_{C1}}{dt} = -q_{C1}^{\frac{5}{3}} + i_{L2} - I_S \\ \frac{di_{L2}}{dt} = 3\sqrt[3]{i_{L2}^2}\left[-q_{C1}^{\frac{1}{3}} - e^{i_{L2}} - E_S\right] \end{cases}$$

Dato il seguente circuito:



1) Ricavare le eq. di stato
2) Calcolare punti di equilibrio e studiare, per $R_2 \in \mathbb{R}$, il variare del tipo di punto!

Eq di stato:



$i_{R1} = \frac{E-V_{C1}}{R_1}$; $i_{R2} = \frac{V_{C1}-V_{C2}}{R_2}$; $i_{R3} = \frac{V_{C2}}{R_3}$;

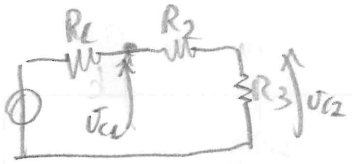
$$i_{C1} = i_{R1} - i_{R2} = \frac{E-V_{C1}}{R_1} - \frac{V_{C1}-V_{C2}}{R_2} = \frac{E}{R_1} - V_{C1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) + \frac{V_{C2}}{R_2}$$

$$i_{C2} = i_{R2} - i_{R3} = \frac{V_{C1}-V_{C2}}{R_2} - \frac{V_{C2}}{R_3} = \frac{V_{C1}}{R_2} - V_{C2}\left(\frac{1}{R_2}+\frac{1}{R_3}\right)$$

$$\rightarrow \underbrace{\begin{bmatrix} \frac{dv_{C1}}{dt} \\ \frac{dv_{C2}}{dt} \end{bmatrix}}_{\underline{\dot{x}}(t)} = \underbrace{\begin{bmatrix} -\frac{1}{C_1}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) & \frac{1}{R_2 C_1} \\ \frac{1}{R_2 C_2} & -\frac{1}{C_2}\left(\frac{1}{R_2}+\frac{1}{R_3}\right) \end{bmatrix}}_{\underline{\underline{A}}} \underbrace{\begin{bmatrix} v_{C1} \\ v_{C2} \end{bmatrix}}_{\underline{x}(t)} + \underbrace{\begin{bmatrix} \frac{E}{R_1} \\ 0 \end{bmatrix}}_{\underline{E}}$$

Due modi di pensare:
- 1: Risolvere $\underline{\dot{x}}(t) = 0$: giusto ma lungo!
- 2: In un circuito alimentato in continua, $C \rightarrow$ aperto, $L \rightarrow$ chiuso } in stato di equilibrio, poiché si va fuori dal transitorio

$\rightarrow$



$$\underline{x}(t) = \begin{cases} v_{C1} = E \cdot \frac{R_2+R_3}{R_1+R_2+R_3} = 8,33\,V \\ v_{C2} = E \cdot \frac{R_3}{R_1+R_2+R_3} = 4,1667\,V \end{cases} \quad Q$$

$$\text{eig}\{\underline{\underline{A}}\} = \text{eig}\begin{bmatrix} -0,07 & 0,02 \\ 0,02 & -0,04 \end{bmatrix} = \underbrace{-0,08 \ ; \ -0,03}_{\text{calcolatrice}} \rightarrow \text{eig}\{\underline{\underline{A}}\} \text{ per } Q$$

$$\text{eigVc} = \begin{bmatrix} -0,07-\lambda_1 & 0,02 \\ 0,02 & -0,04-\lambda_1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \underline{0} \rightarrow \begin{cases} \cancel{0,01x + 0,02y = 0} \\ 0,02x + 0,04y = 0 \rightarrow x = -2y \end{cases}$$

$\lambda_1 = -0,08 \rightarrow x = -2y$ $[1 ; -0,5]$ → autovettore VELOCE [modulo di λ più grosso]
$\lambda_2 = -0,03 \rightarrow 2x = y$ $[1 ; 2]$ → autovettore LENTO

Da chiarire alcune cose!

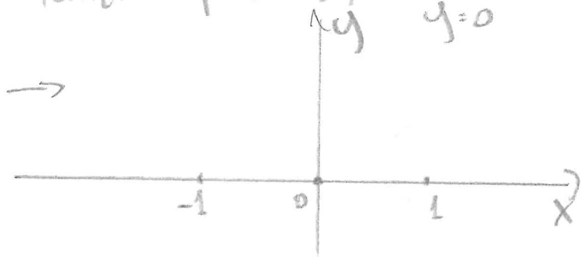
$$\begin{cases} \dfrac{dx}{dt} = -x + x^3 \\ \dfrac{dy}{dt} = -2y \end{cases}$$

$\boxed{2}$ Ricercare i punti di equilibrio e studiarne la stabilità

Punti equilibrio: $-x + x^3 = 0 \rightarrow x = 0,\ (x^2 - 1) = 0 \rightarrow x = \pm 1$

$y = 0$

$\rightarrow$



Calcolo la Hessiana:

$$\underline{\underline{H}} = \begin{bmatrix} \dfrac{\partial f_1}{\partial x} & \dfrac{\partial f_1}{\partial y} \\ \dfrac{\partial f_2}{\partial x} & \dfrac{\partial f_2}{\partial y} \end{bmatrix} = \begin{bmatrix} 3x^2 - 1 & 0 \\ 0 & -2 \end{bmatrix}$$

Sostituiamo nella matrice i punti di equilibrio: $\underline{Q_1} = (0;0)$; $\underline{Q_2} = (1;0)$; $\underline{Q_3} = (-1;0)$

$$H_{Q_1} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \rightarrow \text{eig. neg.} \rightarrow \text{stabile} \quad ; \quad \underline{\underline{H}}_{Q_{2,3}} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} \rightarrow \text{Saddle node}$$

3) $$\left.\begin{aligned} \frac{dx}{dt} &= x(3 - x - 2y) \\ \frac{dy}{dt} &= y(2 - x - y) \end{aligned}\right\} \text{Volterra Lotka model of competition}$$

Stabilità: $\underline{Q_1} = (0;0)$;

$$\begin{cases} 3 - x - 2y = 0 \\ 3 - x - y = 0 \end{cases} \rightarrow x = 3 - 2y \qquad x = 3$$

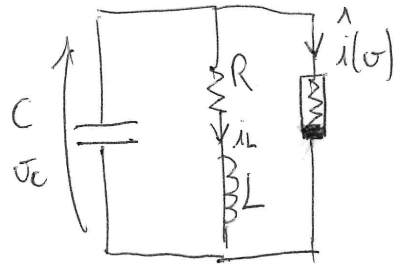
$\cancel{3} - \cancel{3} + 2y - y = 0 \quad y = 0$

$\underline{Q_3} = (3;0)$; $\underline{Q_2} = (0;2)$; $\underline{Q_4} = (1;1)$

$$\underline{\underline{H}} = \begin{bmatrix} 3 - 2x - 2y & -2x \\ -y & 2 - x - 2y \end{bmatrix} \qquad \underline{\underline{H}}_{00} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \quad \underline{\text{instabile!}}$$

Reti non lineari - temi di esame [01/02/2005]

Dato il seguente circuito:



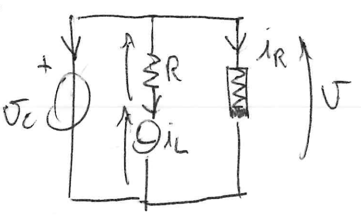
$R = 1\Omega$; $\hat{i}(v) = -2v + v^3$

$C = 1F$;

$L = 1H$;

$\underline{i_2 = -2v_C + v_C^3}$

$\longrightarrow$



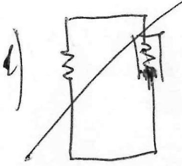
$$\begin{cases} v = v_C; \\ i_C = -i_L - i_R \\ v_L = v_C - Ri_L \end{cases} \Rightarrow \begin{cases} \dfrac{dv_C}{dt} = \dfrac{1}{C}\left[-i_L + 2v_C - v_C^3\right] \\ \dfrac{di_L}{dt} = \dfrac{1}{L}\left[v_C - Ri_L\right] \end{cases}$$

Sostituendo i valori:

$$\begin{cases} \dfrac{dv_C}{dt} = 2v_C - v_C^3 - i_L \\ \dfrac{di_L}{dt} = v_C - i_L \end{cases}$$

$Q_i = (v_C; i_L)$

Punti di equilibrio:

1) 

2) $\begin{cases} 2v_C - v_C^3 - i_L = 0 \quad 2v_C - v_C^3 - v_C = 0 \\ v_C - i_L = 0 \end{cases}$

$\longrightarrow v_C(1 - v_C^2) = 0$

Punti: $\underline{Q_1 = (0;0)}$; $\underline{Q_2 = (1;1)}$; $Q_3 = (-1;-1)$;

$$\underline{\underline{A}} = \begin{bmatrix} 2 - 3v_C^2 & -1 \\ 1 & -1 \end{bmatrix}$$

$\xrightarrow{Q_1} \lambda_i$: punto di sella: $\lambda_i = 1{,}6, -0{,}618$

$\xrightarrow{Q_2} \lambda_1 = \lambda_2^*$: fuoco stabile

$\xrightarrow{Q_3} \lambda_1 = \lambda_2^*$; . . .

Prova di calcolo eigVl:

$$\cancel{\det\left[\underline{\underline{A}}\right]\left|\begin{bmatrix} 2 - 3v_C^2 - \lambda & -1 \\ 1 & -1-\lambda \end{bmatrix}\right| = (2 - 3v_C^2 - \lambda)(-1-\lambda) + 1 = -2 + 3v_C^2 + \lambda - 2\lambda + 3v_C^2\lambda + \lambda^2 + 1}$$

$$\cancel{\longrightarrow \lambda^2 + \lambda(-1 + 3v_C^2) + 1 + 3v_C^2 = 0}$$

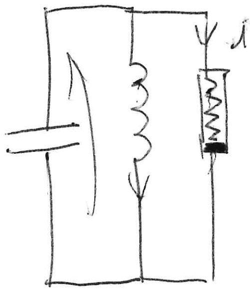
$$\underline{\underline{A}}\Big|_{Q_2} = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \rightarrow \quad (-1-\lambda)^2 + 1 = 0 \rightarrow \lambda^2 + 2\lambda + 2 = 0 \quad \frac{\Delta}{4} = 1 - 2 < 0$$

$$\rightarrow \lambda = \frac{-1 \pm \sqrt{-1}}{1} = -1 \pm j$$

Reti non lineari - 30/01/2008

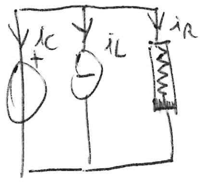
Ricavare le eq. di stato del seguente circuito:

1)



$i_R = -v_R + \frac{1}{3} v_R^3$

$\longrightarrow$



$v_C = v_L$;

$i_C = -i_L - i_R = -i_L - \left(-v_C + \frac{1}{3} v_C^3\right)$

$$\longrightarrow \begin{cases} \dfrac{di_L}{dt} = \dfrac{1}{L} v_C \\ \dfrac{dv_C}{dt} = \dfrac{1}{C}\left[v_C - i_L - \dfrac{1}{3} v_C^3\right] \end{cases}$$

2)
$$\begin{cases} \dot{x}_1 = -x_1 + x_1 x_2 \nearrow x_1(x_2-1) \\ \dot{x}_2 = x_1 + x_2 \end{cases} \longrightarrow x_1 = 0,\ x_2 = 0$$

$x_2 = 1,\ x_1 = -1$

$$\underline{\underline{A}} = \begin{bmatrix} x_2 - 1 & x_1 \\ 1 & 1 \end{bmatrix}$$

$\underline{\underline{A}}\Big|_{0,0} = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}$ $\left.\begin{matrix} \lambda_1 = 1 \\ \lambda_2 = -1 \end{matrix}\right\}$ Punto di Sella;

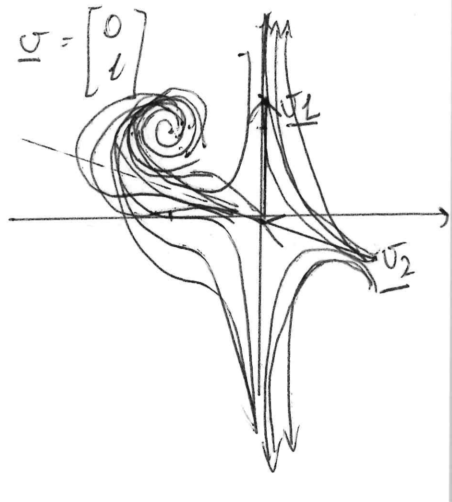
$\underline{\underline{A}}\Big|_{-1,1} = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$

$\rightarrow$ Autovettori λ_1: $\begin{bmatrix} -1-1 & 0 \\ 1 & -1+1 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix}$ $\quad -v_1 \cdot 2 = 0$, v_2 no condizioni $\rightarrow \begin{bmatrix} v_1 = 0, \\ v_2 = 1 \end{bmatrix} \rightarrow \underline{v} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

a caso

λ_2: $\begin{bmatrix} -1+1 & 0 \\ 1 & 1+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 2 \end{bmatrix}$ $\quad v_1 = -2v_2$

$\rightarrow [1, -0{,}5]$

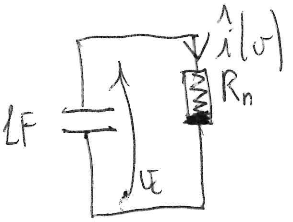


$\underline{A}\Big|_{-1,1} = \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} -\lambda & -1 \\ 1 & 1-\lambda \end{bmatrix} \Rightarrow -\lambda + \lambda^2 + 1 = 0 \quad \lambda = \frac{1}{2} \pm j\sqrt{\frac{3}{4}}$

$\rightarrow$ FUOCO INSTABILE

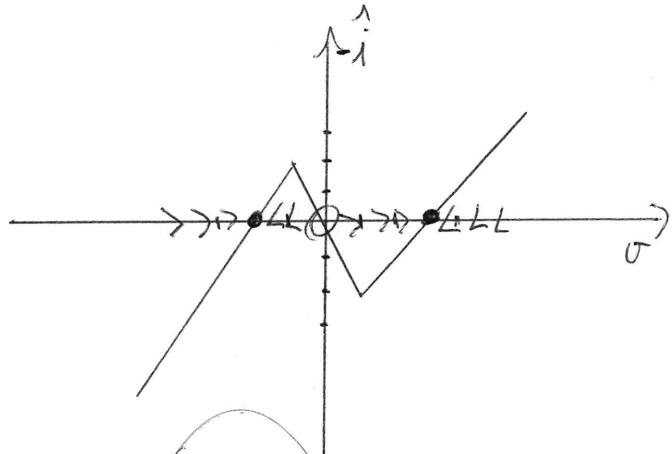
Reti non lineari - Tema di esame 17/02/2005

Si consideri il seguente circuito:



Dove:

$$\hat{i}(v) = \begin{cases} i = 2v+4, & v \le 1 \\ i = -2v, & -1 \le v \le 2 \\ i = v-6, & v \ge 2 \end{cases}$$



$i_c = i_v$;

$$i_c = \dots \longrightarrow \begin{cases} \frac{dv_c}{dt} = -(2v+4), & v \le 1 \\ \frac{dv_c}{dt} = -(-2v), & -1 \le v \le 2 \\ \frac{dv_c}{dt} = -(v-6) & v \ge 2 \end{cases} \longrightarrow$$

3 punti di equilibrio:

$$\begin{cases} v = 0 \to \text{instabile} \\ v = -2 \to \text{stabile} \\ v = 6 \to \text{stabile} \end{cases}$$

Cosa c'é da dire su esistenza e unicitá? Studiamo la lipschitzianitá: dire lipschitz. significa dire "é possibile comprendere il grafico tra due rette $\pm L x$, L pendenza FINITA"; poiché $|i(v_2) - i(v_1)| \le |2v|$, ossia esistono rette la cui pendenza finita é in grado di maggiorare il cambio di pendenza, la funzione é lipschitziana ma non derivabile. La sol. ∃ ed é unica.

Se f é continua c'é una o piú soluzioni, se é lip. é UNICA!

3) L'origine respinge, quindi: $D_{-2} = v < 0$
$D_6 = v > 0$

4) Si procede cosí: si studia in quale punto ci si trova, e si prende la corrispondente eq. di stato; poiché $v(0) = 0{,}1V$, $-1 \le v(0) \le 2v$, siamo con $\frac{dv_c}{dt} = 2v$, e il valore di regime é 6V (punto di equilibrio); risolvendo il problema:

$\frac{dv_c}{dt} = 2v \to v = Ae^{2t}$; per $t=0$, $Ae^{2t} = 0{,}1V \to A = 0{,}1V$;

da $t=0$ a $t=t^*$, si passa da 0,1 V a 2V (punto v^* di fine intervallo); $2 = 0{,}1e^{2t^*}$; $e^{2t^*} = \frac{2}{0{,}1}$;

$t^* = \frac{\ln\left(\frac{2}{0{,}1}\right)}{2} = \frac{\ln(20)}{2}$.

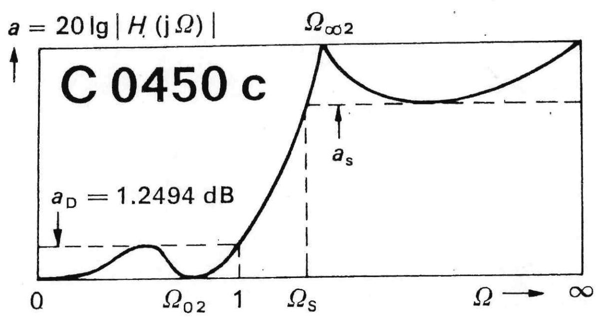
Da $v>2$, si cambia musica: da qua potremo sul serio raggiungere $v=6$, ma "con calma":

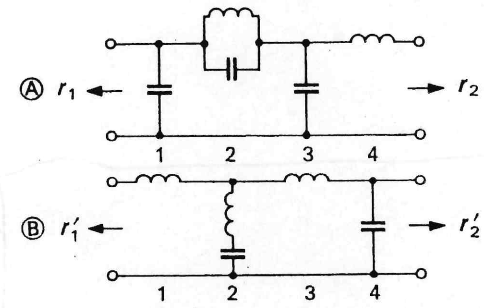
$\frac{dv_c}{dt} = 6 - v$, $v \ge 2$; usiamo la linearitá della derivata, e otteniamo: v_c = SOVRAPP. EFFETTI:

$\frac{dv_c}{dt} = 6 \to v_c = B$; $\frac{dv_c}{dt} = -v \to v_c = Ae^{-t}$; A regime andiamo a 6, ma la funzione parte da $v=2$, $t = t^*$; per $t = t^*$, $v_c(t^*) = 2$;

$2 = v_c(t^*) = 6 + Ae^0 \to A = -4$

$\to v_c(t) = 6 - 4e^{-(t-t^*)}$





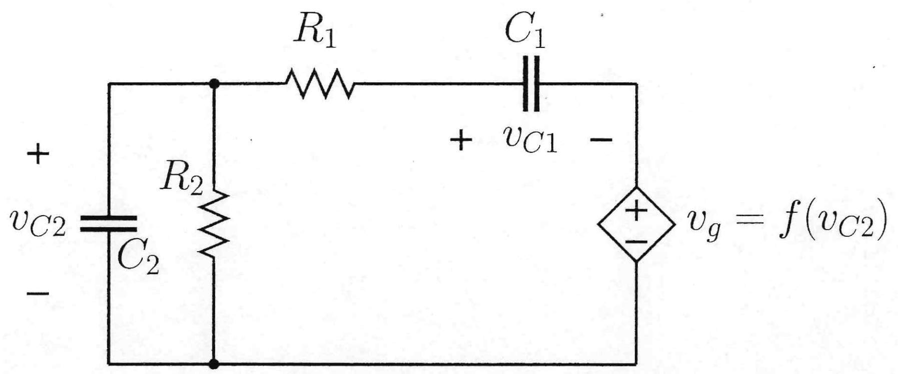
$$H(p) = C\,\frac{\prod_{\nu=1}^{2}(p^2 - 2\alpha_\nu p + \gamma_\nu)}{p^2 + \Omega_{\infty 2}^2}$$

$$\gamma_\nu = \alpha_\nu^2 + \beta_\nu^2$$

Θ	Ω_s	a_s dB	Ⓐ ν / Ⓑ ν	Ⓐ $r_1 = 1$: $c_{2\nu-1}$ / Ⓑ $l_{2\nu-1}$, $r_1' = 1$	Ⓐ $l_{2\nu}$ / Ⓑ $c_{2\nu}$	Ⓐ $r_2 = 1$: $c_{2\nu}$ / Ⓑ $l_{2\nu}$, $r_2' = 1$	Ⓐ $r_1 = \infty$: $c_{2\nu-1}$ / Ⓑ $l_{2\nu-1}$, $r_1' = 0$	Ⓐ $l_{2\nu}$ / Ⓑ $c_{2\nu}$	Ⓐ $r_2 = 1$: $c_{2\nu}$ / Ⓑ $l_{2\nu}$, $r_2' = 1$	Ⓐ $r_1 = 1$: $c_{2\nu-1}$ / Ⓑ $l_{2\nu-1}$, $r_1' = 1$	Ⓐ $l_{2\nu}$ / Ⓑ $c_{2\nu}$	Ⓐ $r_2 = 0$: $c_{2\nu}$ / Ⓑ $l_{2\nu}$, $r_2' = \infty$	$\Omega_{\infty 2\nu}$	$\Omega_{0\nu}$	$-\alpha_\nu$	$\pm\beta_\nu$	C
P			1	0.667159	1.610665		1.334318	1.374788						0.0000000000	1.0598770853	0.4390154632	0.577350269
			2	1.610665	0.667159		0.943506	0.333580						0.0000000000	0.4390154632	1.0598770853	
T			1	1.671794	1.551771		1.614017	1.549622						0.0000000000	0.4474504643	0.3272912223	3.365043970
			2	1.551771	1.671794		1.609548	0.835897						0.9101797211	0.1507092082	0.9717164010	
12	5.616886431	85.5	1	1.652475	1.524734	0.017261	1.599057	1.522906	0.017281	0.818274	1.578126	0.016677	6.164147748	0.0000000000	0.4500715899	0.3305558568	125.883983681
			2	1.541804	1.669544		1.595222	0.834772		1.539404	1.616153			0.9112186568	0.1488941877	0.9725153887	
13	5.188530973	82.7	1	1.649114	1.520039	0.020299	1.596458	1.518264	0.020323	0.815202	1.572663	0.019620	5.692923261	0.0000000000	0.4505304345	0.3311286804	107.081378290
			2	1.540073	1.669149		1.592730	0.834575		1.537636	1.616525			0.9114001252	0.1485771147	0.9726545207	
14	4.821651409	80.1	1	1.645483	1.514966	0.023594	1.593651	1.513248	0.023621	0.811880	1.566761	0.022815	5.289238737	0.0000000000	0.4510272645	0.3317493684	92.161999365
			2	1.538204	1.668721		1.590036	0.834361		1.535728	1.616927			0.9115964789	0.1482340164	0.9728049208	
15	4.503953976	77.7	1	1.641580	1.509517	0.027151	1.590636	1.507859	0.027181	0.808307	1.560418	0.026265	4.939586997	0.0000000000	0.4515623829	0.3324184053	80.125717615
			2	1.536196	1.668260		1.587141	0.834130		1.533682	1.617359			0.9118078047	0.1478647372	0.9729666195	
16	4.226218458	75.4	1	1.637406	1.503691	0.030971	1.587412	1.502096	0.031004	0.804482	1.553634	0.029976	4.633837719	0.0000000000	0.4521361194	0.3331363167	70.274813658
			2	1.534050	1.667764		1.584044	0.833882		1.531496	1.617821			0.9120341960	0.1474691088	0.9731396497	
17	3.981394206	73.3	1	1.632958	1.497488	0.035061	1.583979	1.495959	0.035096	0.800403	1.546409	0.033951	4.364244243	0.0000000000	0.4527488303	0.3339036711	62.110534983
			2	1.531764	1.667234		1.580743	0.833617		1.529173	1.618313			0.9122757536	0.1470469501	0.9733240457	
18	3.763997479	71.3	1	1.628237	1.490908	0.039423	1.580338	1.489447	0.039461	0.796070	1.538742	0.038197	4.124781448	0.0000000000	0.4534008998	0.3347210808	55.268698332
			2	1.529340	1.666670		1.577240	0.833335		1.526712	1.618836			0.9125325854	0.1465980668	0.9735198439	
19	3.569699103	69.4	1	1.623243	1.483951	0.044063	1.576487	1.482560	0.044104	0.791480	1.530631	0.042719	3.910692709	0.0000000000	0.4540927397	0.3355892033	49.478359448
			2	1.526778	1.666070		1.573533	0.833035		1.524115	1.619389			0.9128048070	0.1461222506	0.9737270822	
20	3.395035834	67.6	1	1.617973	1.476615	0.048986	1.572428	1.475298	0.049030	0.786633	1.522077	0.047523	3.718172782	0.0000000000	0.4548247926	0.3365087432	44.534567981
			2	1.524076	1.665434		1.569622	0.832717		1.521381	1.619972			0.9130925413	0.1456192795	0.9739457999	
21	3.237204182	65.8	1	1.612428	1.468902	0.054198	1.568159	1.467659	0.054244	0.781527	1.513077	0.052616	3.544141280	0.0000000000	0.4555975308	0.3374804539	40.279977878
			2	1.521235	1.664762		1.565505	0.832381		1.518513	1.620586			0.9133959193	0.1450889171	0.9741760378	
22	3.093910472	64.2	1	1.606607	1.460810	0.059705	1.563680	1.459643	0.059753	0.776160	1.503632	0.058005	3.386077936	0.0000000000	0.4564114588	0.3385051392	36.592169792
			2	1.518256	1.664053		1.561182	0.832027		1.515509	1.621231			0.9137150801	0.1445309121	0.9744178378	
23	2.963260016	62.6	1	1.600508	1.452339	0.065514	1.558992	1.451249	0.065563	0.770530	1.493740	0.063698	3.241900840	0.0000000000	0.4572671146	0.3395836560	33.374744421
			2	1.515137	1.663306		1.556653	0.831653		1.512373	1.621905			0.9140501707	0.1439449983	0.9746712429	
24	2.843673992	61.1	1	1.594131	1.443489	0.071631	1.554094	1.442477	0.071681	0.764634	1.483400	0.069704	3.109875113	0.0000000000	0.4581650704	0.3407169159	30.550957419
			2	1.511880	1.662522		1.551916	0.831261		1.509104	1.622611			0.9144013469	0.1433308935	0.9749362970	

01JES Filtri e reti non lineari 2/02/2009

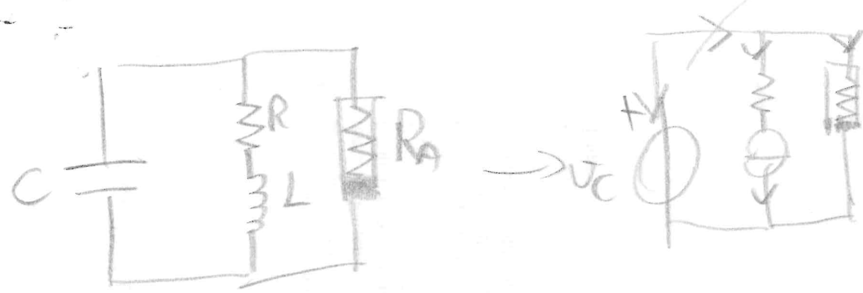
1. Si scrivano le equazioni di stato per il circuito indicato in figura, ove il generatore di tensione non lineare controllato in tensione è descritto dalla relazione costitutiva $v_g = f(v_{C2})$.



2. Un circuito non lineare è descritto dalle seguenti equazioni di stato

$$\begin{aligned} \frac{dx_1}{dt} &= x_1 x_2 - 1 \\ \frac{dx_2}{dt} &= 16x_1 - x_2^3 \end{aligned}$$

Si determinino i punti d'equilibrio e il loro tipo (nodo, fuoco, sella ecc.). Infine si tracci un andamento qualitativo del diagramma delle fasi.



$i = -2v + v^3$

$$\to \begin{cases} i_C = -i_L - \left(-2v_C + v_C^3\right) \\ v_L = v_C - R i_L \end{cases} \to \quad 2v_C - i_L - v_C^3 = 0 \qquad v_C\left(2 - v_C^2\right) - i_L = 0$$

$v_C - i_L = 0$

$\downarrow (0,0),\ (1;1);\ (-1;-1)$

$$\underline{\underline{A}} = \begin{bmatrix} -3v_C^2 + 2 & -1 \\ 1 & -1 \end{bmatrix}$$

$$\to \underline{\underline{A}}_{00} = \begin{bmatrix} 2 & -1 \\ 1 & -1 \end{bmatrix} \to (2-\lambda)(-1-\lambda) + 1 = -2 - 2\lambda + \lambda + \lambda^2 + 1 = 0$$

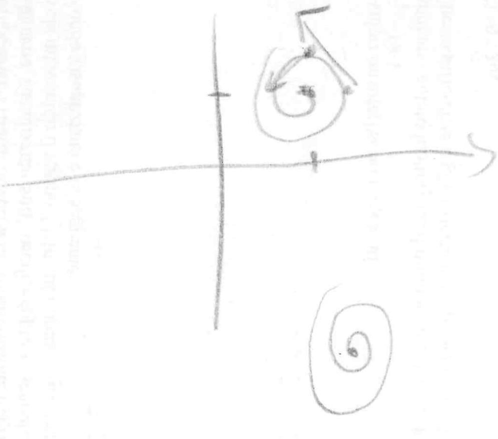
$$\to \lambda^2 - \lambda - 1 = 0 \to 1 + 4 \to \frac{1 \pm \sqrt{5}}{2}$$

$$\underline{\underline{A}}_{11} \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} \to (-1-\lambda)^2 + 1 = \lambda^2 + 2\lambda + 1 + 1 = > \frac{\Delta}{4} = 1 - 2 = -1$$

$\updownarrow \quad \lambda = -1 \pm j$

fuoco stabile

$$\underline{\underline{A}}_{-1-1} = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}$$



$$\begin{cases} \dot{x} = 2v_C - i_L - v_C^3 \\ \dot{y} = v_C - i_L \end{cases}$$

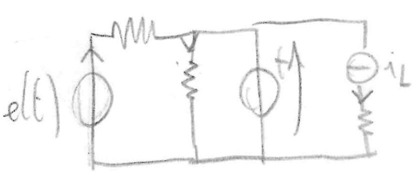
$\dot{x} = 2 - 1{,}5^3 - 1 = -0{,}5$ $\quad v_C = 1{,}5$

$\dot{y} = 1 - 1{,}5 = -0{,}5$ $\quad i_L = 1$

$3 - 1 - 3{,}373$

$-1{,}375 - \dot{x}$

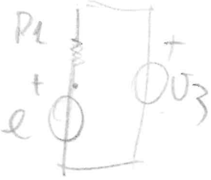
$1 - 1{,}5 = +0{,}5$



$i_{R2} = \frac{v_{C3}}{R_2}$

$v_L = v_3 - i_L R_5$

$i_{R1} = \frac{-v_3 + e}{R_1}$

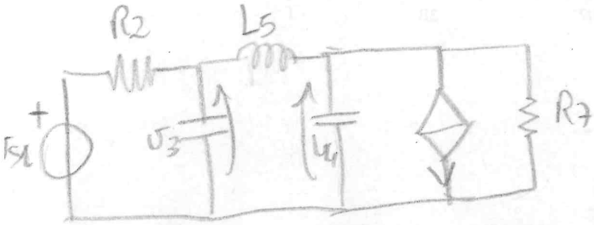


$$i_{C1} = i_{R1} - i_{R2} - i_L = \frac{e - v_3}{R_1} - \frac{v_3}{R_2} - i_L$$

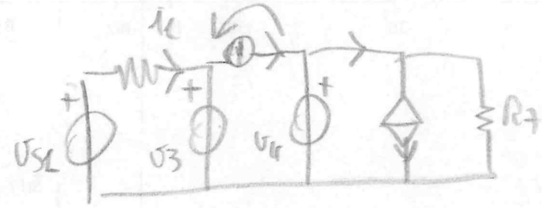
$$v_L = v_3 - i_L R_5$$

$$\rightarrow \begin{bmatrix} \frac{dv_c}{dt} \\ \frac{di_L}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{(G_1+G_2)}{C} & -\frac{1}{C} \\ \frac{1}{L} & -\frac{R_5}{L} \end{bmatrix} \begin{bmatrix} v_c \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{e}{R_1} \\ 0 \end{bmatrix}$$

$$\begin{cases} \frac{dv_1}{dt} = \frac{1}{C}\left[\frac{e}{R_1} - v_3\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - i_L\right] \\ \frac{di_L}{dt} = \frac{1}{L}\left[v_3 - i_L R_5\right] \end{cases}$$



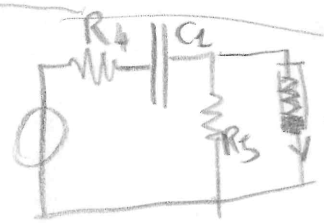
$\longrightarrow$

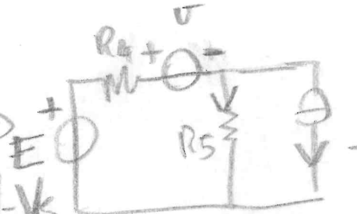


$$\longrightarrow i_1 = \frac{v_{S1} - v_3}{R_2} \;;\quad i_3 = i_1 - i_L \;;\quad v_L = v_3 - v_4 \;;\quad i_7 = \frac{v_4}{R_7} \;;$$

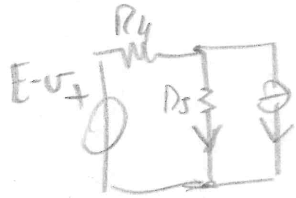
$$i_4 = i_L - (i_7 + g_m v_3) \;;$$

$$\rightarrow \begin{aligned} \frac{dv_3}{dt} &= \frac{1}{C}\left[\frac{E - v_3}{R_2} - i_L\right] \\ \frac{dv_4}{dt} &= \frac{1}{C}\left[i_L - \frac{v_4}{R_7} - g_m v_3\right] \\ \frac{di_L}{dt} &= \frac{1}{L}\left[v_3 - v_4\right] \end{aligned} \longrightarrow \begin{bmatrix} \frac{dv_3}{dt} \\ \frac{dv_4}{dt} \\ \frac{di_L}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_2 C} & 0 & -\frac{1}{C} \\ -\frac{g_m}{C} & -\frac{1}{CR_7} & \frac{1}{C} \\ \frac{1}{L} & -\frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} v_3 \\ v_4 \\ i_L \end{bmatrix} + \begin{bmatrix} \frac{E}{CR_2} \\ 0 \\ 0 \end{bmatrix}$$





$\longrightarrow$



$$\frac{\frac{E - v}{R_4} + 0 - i_L}{\frac{1}{R_4} + \frac{1}{R_5}} = \frac{R_4 R_5}{R_4 + R_5}\left[\frac{E - v}{R_4} - i_L\right] = V_S \;;\quad i_C = \frac{V_S}{R_5} + i_L \;;\quad V_L = V_S$$

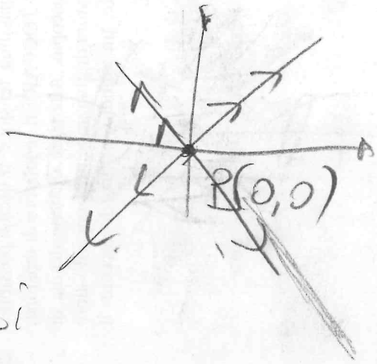
$P(0,0)$

$$J(0,0) = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix} \underset{=}{\text{autoval}} \begin{vmatrix} 3-\lambda & 0 \\ 0 & 2-\lambda \end{vmatrix} = (3-\lambda)(2-\lambda) = 0$$

$\lambda_1 = 3$

$\lambda_2 = 2$

$\lambda_1 . \lambda_2 > 0$ reali $\Rightarrow$ NODO INSTABILE



Calcolo autovettori

Autovettore dell'autovalore più piccolo (2)

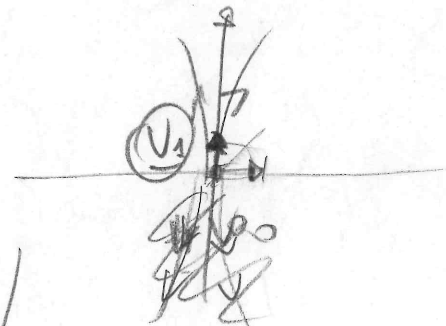
$$\begin{pmatrix} \cancel{3} & \cancel{0} \\ \cancel{0} & \cancel{2} \end{pmatrix} \begin{pmatrix} 3-2 & 0 \\ 0 & 2-2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} 1\, V_1 = 0 \\ 0\, V_2 = 0 \end{cases}$$

$v_1 = 0$

$v_2 =$ qualsiasi $= 1$

$\underline{V_1} = (v_1, v_2) = (0, 1)$



$P(3,0)$

$$\begin{pmatrix} 3-6 & -6 \\ 0 & 2-3 \end{pmatrix} = \begin{pmatrix} -3 & -6 \\ 0 & -1 \end{pmatrix}$$

$$\frac{-4 \pm 2}{2} < \begin{matrix} -3 \\ -1 \end{matrix}$$

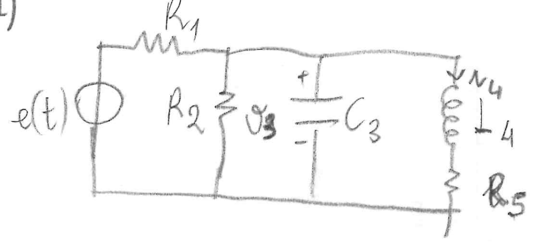
$$\lambda_{1,2} = \frac{-4 \pm \sqrt{16-12}}{2}$$

$\lambda^2 + 4\lambda + 3 = 0$

autovalori $\begin{vmatrix} -3-\lambda & -6 \\ 0 & -1-\lambda \end{vmatrix} = (-3-\lambda)(-1-\lambda) + \cancel{6} = 3 + 3\lambda + \lambda + \lambda^2 + \cancel{6} = 0$

ESERCITAZIONE

1)



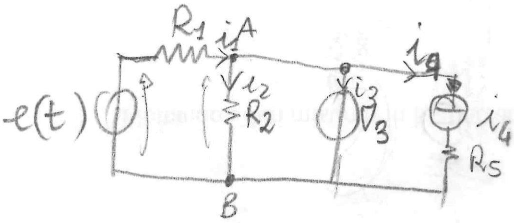
$R_1 = 1k\Omega$

$R_2 = R_5 = 2k\Omega$

$C_3 = 1mF$ $\quad L_4 = 1mH$

$e(t) = 5V$

Eq di stato



$\longrightarrow Z$

Valutare le coniugate (i_3) e (V_5)

$i_1 - i_2 - i_3 - i_4 = 0$

$i_3 = i_1 - i_2 - i_4$

$i_2 = \frac{V_3}{R_2} \qquad i_1 = \frac{e(t) - V_3}{R_1}$

$i_3 = \frac{e(t) - V_3}{R_1} - \frac{V_3}{R_2} - i_L = \frac{e(t)}{R_1} - \frac{V_c}{R_1} - \frac{V_c}{R_2} - i_L \quad i_C$

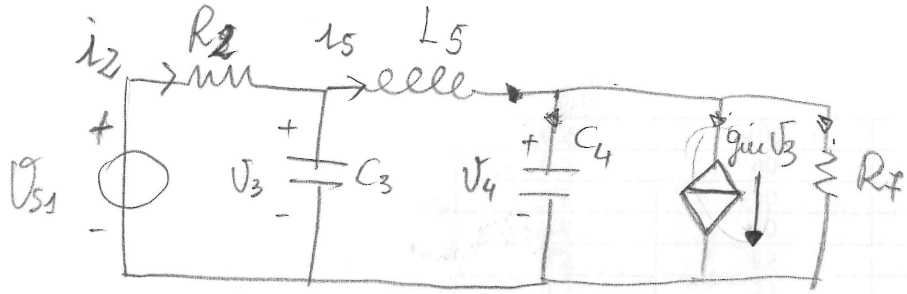
$V_L = V_3 - i_4 R_5 = V_C - i_L R_5 \quad V_L$

$i_C = C\frac{dv_C}{dt} = \frac{e(t)}{R_1} - \frac{V_C}{R_1} - \frac{V_C}{R_2} - i_L = C\frac{dv_C}{dt}$

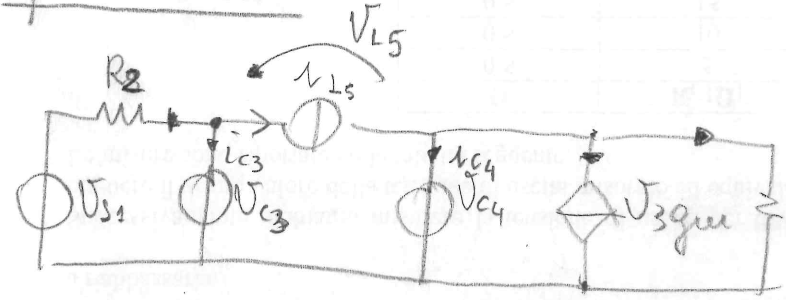
$V_L = L\frac{di(t)}{dt} = V_C - i_L R_5 = L\frac{d i_L(t)}{dt}$

$$\begin{cases} \frac{dv_C(t)}{dt} = \frac{1}{C}\left[\frac{e(t)}{R_1} - \frac{V_C}{R_1} - \frac{V_C}{R_2} - i_L\right] \\ \frac{di(t)}{dt} = \frac{1}{L}\left[V_C - i_L R_5\right] \end{cases}$$

$$\begin{bmatrix} \frac{dV_C}{dt} \\ \frac{di}{dt} \end{bmatrix} \begin{matrix} \frac{1}{C} \\ \frac{1}{L} \end{matrix} \begin{bmatrix} -1 & -\frac{V_C}{R_1} - \frac{V_C}{R_2} \\ -R_5 & +1 \end{bmatrix} \begin{bmatrix} i_L \\ V_C \end{bmatrix} + \begin{bmatrix} \frac{e(t)}{R_1} \\ 0 \end{bmatrix}$$



Eq di stato



$$i_{C3} = i_1 - i_{L5} \qquad i_1 = \frac{V_{S1} - V_{C3}}{R_2} \qquad i_{C3} = \frac{V_{S1}}{R_2} - \frac{V_{C3}}{R_2} - i_{L5}$$

$$V_{LC} = V_{C3} - V_{C4}$$

$$i_{C4} = i_{L5} - V_3 g_m - \frac{V_{C4}}{R_7} = i_{L5} - V_{C3} g_m - \frac{V_{C4}}{R_7}$$

$$C_3 \frac{dV_{C3}}{dt} = \frac{V_{S1}}{R_2} - \frac{V_{C3}}{R_2} - i_{L5}$$

$$C_4 \frac{dV_{C4}}{dt} = i_{L5} - V_{C3} g_m - \frac{V_{C4}}{R_7}$$

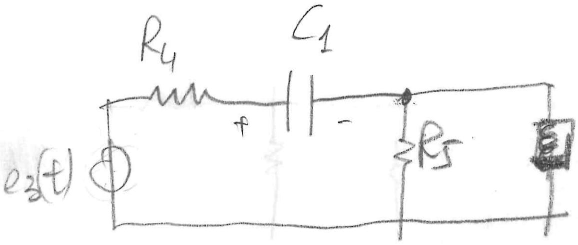
$$L_5 \frac{di_{L5}}{dt} = V_{C3} - V_{C4}$$

$$C_3 \frac{dV_{C3}}{dt} = -\frac{V_{C3}}{R_2} - i_{L5} + \frac{V_{S1}}{R_2}$$

$$C_4 \frac{dV_{C4}}{dt} = -V_{C3} g_m - \frac{V_{C4}}{R_7} + i_{L5}$$

$$L_5 \frac{di_{L5}}{dt} = V_{C3} - V_{C4}$$

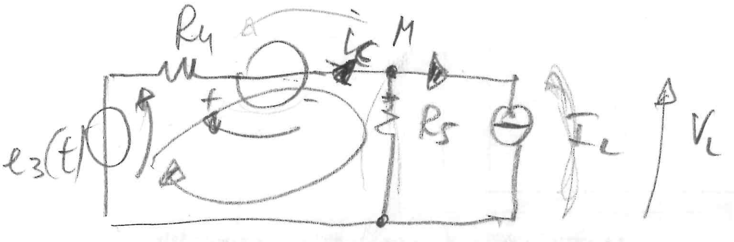
OK



$v_2 = g_2(\varphi_2)$

$$\boxed{\varphi_2 = i_L L}$$

$$v_L = L\frac{di_L}{dt} = \frac{d\varphi_2}{dt}$$



$$V_M = \frac{\frac{e_3(t)-V_C}{R_4} - i_L}{\frac{1}{R_4}+\frac{1}{R_5}} = \frac{\frac{e_3(t)-V_C-R_4 i_L}{R_4}}{\frac{R_5+R_4}{R_4 R_5}} = \frac{(e_3(t)-V_C-R_4 i_L)R_5}{R_5+R_4}$$

$$i_C = \frac{e(t) - V_M - V_C}{R_4} = \left[e_3(t) - \frac{(e_3(t)+V_C-R_4 i_L)R_5}{R_4+R_5} - V_C\right]\frac{1}{R_4}$$

$$i_C = \frac{e_3(t)R_4 + \cancel{e_3(t)R_5} - \cancel{e_3(t)R_5} - V_C R_5 - R_4 R_5 i_L - V_C(R_4+R_5)}{(R_4+R_5)R_4} = \frac{e_3(t)R_4 - V_C R_5 - R_4 R_5 i_L - V_C(R_4+R_5)}{(R_4+R_5)R_4}$$

$$\frac{V_C(R_4+R_5)}{(R_4+R_5)R_4}$$

$\underline{V_L = V_M}$

$$\cancel{\frac{-V_C R_5}{(R_4+R_5)R_4}} - \frac{V_C R_4}{(R_4+R_5)R_4} + \cancel{\frac{V_C R_5}{(R_4+R_5)R_4}}$$

$$\begin{cases} C\dfrac{dV_C}{dt} = i_C = \dfrac{e_3(t)}{R_4+R_5} - \cancel{\dfrac{V_C R_5}{(R_4+R_5)R_4}} - \dfrac{R_4 R_5 i_L}{(R_4+R_5)R_4} - \cancel{\dfrac{V_C(R_4+R_5)}{(R_4+R_5)R_4}} - \dfrac{V_C}{R_4+R_5} \\ \dfrac{d g_2(i_L)}{dt} = v_L = \dfrac{e_3(t)R_5}{R_5+R_4} - \dfrac{V_C R_5}{R_4+R_5} - \dfrac{R_4 i_L}{R_4+R_5}R_5 \end{cases}$$

$C = \frac{dq}{dV}$ $V = L = d$

q_{C_1} φ_{L_2}

$$\begin{cases} I_{C_1} = -I_{RA} - I_2 + I_{L_2} \\ V_{L_2} = -(V_{RB} + V_{C_1} + E) \end{cases}$$

fun q_{C_1} e φ_{L_2}

$q_{C_1} = V_{C_1}^3$ $I_{RA} = V_A^5$

$\varphi_{L_2} = i_{L_2}^{\frac{1}{3}}$ $V_B = e^{ib}$

$$\begin{cases} I_{C_1} = -V_A^5 - I_2 + I_{L_2} \\ V_{L_2} = -(e^{ib} + V_{C_1} + E) \end{cases}$$

$$C\frac{dV_C}{dt} = -V_A^5 - I_2 + I_{L_2}$$ $C\,dV_C \to dq$

$$L\frac{di}{dt} = -(e^{ib} + V_{C_1} + E)$$ $\to d\varphi$ $\frac{d\varphi}{dt} = V_L$ $\varphi_L = i_L^3$ $i_L = \varphi_L^{\frac{1}{3}}$

$$\frac{dq}{dt} = -V_{C_1}^5 - I_2 + I_{L_2}$$ $q_{C_1} = V_{C_1}^3$ $V_{C_1} = q^{\frac{1}{3}}$

$$\frac{dq}{dt} = -q^{\frac{5}{3}} - I_2 + \varphi_{L_2}^{3}$$

$$\frac{d\varphi}{dt} = -(e^{ib} + q^{\frac{1}{3}} + E)$$